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Axial Anomaly and the Nucleon Spin

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Abstract

In this letter we have taken a particular Lagrangian, which was introduced to resolve $U(1)$ problem, as an effective QCD Lagrangian, and have derived a formula of the quark content of the nucleon spin. The difference between quark content of the proton ($\Delta\Sigma_p$) and that of the neutron ($\Delta\Sigma_n$) is evaluated by this formula. Neglecting the higher-order isospin corrections, this formula can reduce to Efremov's results in the large N_c limit.

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1 Introduction

The EMC data [1] taken in conjunction with the Bjorken sum rule [2] imply that the total spin carried by the quarks in the polarized proton amounts to only about $\frac{1}{8}$ of the spin of the proton. One possible way to explain these surprising results is to take into account a quantum effect known as the “axial anomaly” of QCD. The anomaly makes it possible for spin carried by gluons to mix with spin carried by quarks, thus modifying the structure of the quark sea. Effectively, if the gluon polarization is big, *i.e.* if the amount of spin carried by the gluon is large and positive, the fraction of the nucleon spin carried by quarks will appear to be smaller than it really is. Thus the existence of a large anomaly effect would explain the smallness of the apparent quark contribution to the proton spin. Six years ago, Cheng and Li [3] suggested, looking at the anomaly effect of the $U(1)$ Ward identity,

$$\partial_\nu J_\nu^5 = \sum_i 2m_i \bar{q}_i i\gamma_5 q_i + \partial_\nu \widetilde{K}_\nu , \quad (1)$$

where J_ν^5 is the axial-vector singlet quark current, and \widetilde{K}_ν is the topological current. From Eq. (1), there seems to be a natural separation of the current matrix element into the quark contribution (the first term), which is hoped to yield the naive quark-model result, and the gluon anomaly contribution (the second). However, the calculated gluon contribution is opposite in sign to what one had expected in the parton model [4]. Actually, in any covariant gauge

$$\begin{aligned} \langle p' | J_\nu^5 | p \rangle &= \bar{u}(p') [\gamma_\nu \gamma_5 G_1(q^2) + q_\nu \gamma_5 G_2(q^2)] u(p) , \\ \langle p' | \widetilde{K}_\nu | p \rangle &= \bar{u}(p') [\gamma_\nu \gamma_5 \widetilde{G}_1(q^2) + q_\nu \gamma_5 \widetilde{G}_2(q^2)] u(p) . \end{aligned} \quad (2)$$

where $q = p - p'$, \bar{u} and u are the polarized proton wave functions, and $G_i(\widetilde{G}_i)$'s are form factors. Above expressions can be written in the form, when $p' = p$,

$$\begin{aligned} \langle p' | J_\nu^5 | p \rangle &= 2M_N S_\nu G_1(0) , \\ \langle p' | \widetilde{K}_\nu | p \rangle &= 2M_N S_\nu \widetilde{G}_1(0) , \end{aligned} \quad (3)$$

where M_N is the nucleon mass, S_ν its spin four-vector. And according to definition of Efremov *et. al.* [5], the contributions parallel to it are, in terms of quark and gluon distributions¹,

$$G_1(0) = \Delta\Sigma - \Delta\tilde{g} \quad \text{and} \quad \tilde{G}_1(0) = -\Delta\tilde{g} \quad , \quad (\Delta\tilde{g} = \frac{\alpha_s}{2\pi} N_f \Delta g) \quad . \quad (4)$$

Thus, the calculated gluon distribution is large and positive. However, they neglected the large isospin violation and assumed that $\Delta m_{\eta'} g_{QNN} \ll g_{\eta'NN} = 6.3$ in the final calculation. We note the following two points. First, the term proportional to $(m_u - m_d)$ is neglected in Ref. [5], due to it leads to higher-order isospin corrections. But, according to Ref. [3], the size of the term contributing to $\Delta\Sigma_p$ is about 0.38. Neglecting thus large contribution, the obtained results are not satisfactory. Second, Kochelev [7] has shown that the contribution of the π^0 -ghost mixing to the charge symmetry breaking (CSB) may be significant, and the value of CSB is determined by the mass difference of d - and u -quarks and the ghost-nucleon coupling constant g_{QNN} . If we determine the value of CSB in the different methods, then the ghost-nucleon coupling constant g_{QNN} can still be obtained. Therefore, the assumption of $\Delta m_{\eta'} g_{QNN} \ll g_{\eta'NN}$ is not always necessary.

In this letter we take aim at solving above two questions. The paper organized as follows. In Section 2, we will start from a particular Lagrangian, and derive a formula of the quark content of nucleon spin. In Section 3, the difference between $\Delta\Sigma_p$ and $\Delta\Sigma_n$ is evaluated. Neglecting the higher-order corrections, we found it comparable with the value of Ref. [5]. In Section 4, the value of the ghost-nucleon coupling constant is obtained by introducing charge symmetry breaking in the pion-nucleon coupling. Finally, discussions and conclusions are given in Section 5.

¹Here we would like to remind the readers that the definition of quark and gluon distribution function is renormalization scheme dependent [6]. Based on the gauge-invariant operator product expansion, the first moment of the flavor-singlet polarized distribution function is given by $\Delta\Sigma(=G_1(0))$, which is equal to $\Delta\Sigma - \Delta\tilde{g}$ in this paper, as shown in (4).

2 The effective Lagrangian and a formula of the quark content in the nucleon spin

The longstanding $U(1)$ problem (including not only η mass problem, but also problem of $\eta \rightarrow \pi^+\pi^-\pi^0$ decay) can be consistently resolved in the following effective Lagrangian, where $U(1)$ anomaly is taken into account as $O(\frac{1}{N_c})$ effect [8] :

$$\mathcal{L} = \frac{F_\pi^2}{16} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \sum_a^{0,3,8} c^a u^a + \frac{1}{2F_s^2 \widetilde{m}^2} (\partial^\mu \widetilde{K}_\mu)^2 - \frac{1}{F_s} (\partial^\mu \widetilde{K}_\mu) S, \quad (5)$$

with

$$U = \exp \left[i \frac{2}{F_\pi} (\lambda^0 S + \lambda^a \pi^a) \right], \quad u^a = \frac{1}{4} \text{Tr} [\lambda^a (U + U^\dagger)], \quad (6)$$

where λ^a are the usual Gell-Mann matrices, F_π is the pion decay constant with input value of $F_\pi = 186.4$ MeV, and $F_s = \sqrt{\frac{3}{2}} F_\pi \sim O(\sqrt{N_c})$, and π^a (S) is the flavor octet (singlet) pseudoscalar field. The explicit $SU_f(3)$ breaking is represented by $(c^0, c^8, c^3) = (\frac{1}{4} \sqrt{\frac{3}{2}} F_\pi^2 m_{N_s}^2, -\frac{F_\pi^2}{2\sqrt{3}} (m_K^2 - m_\pi^2), \frac{1}{4} F_\pi^2 \delta m^2)$ with $m_{N_s}^2 = \frac{1}{3} (2m_K^2 + m_\pi^2)$ and $\delta m^2 = m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2$. The last two terms in Eq. (5) contain an axial vector ghost field \widetilde{K}_μ , which add a mass $\widetilde{m}^2 = m_{\eta'}^2 + m_\eta^2 - 2m_K^2 \sim O(\frac{1}{N_c})$ to the $U(1)$ Nambu-Goldstone boson S . Here \widetilde{K}_μ in Eq. (5) should be identified with a non-perturbative realization of the topological current in QCD

$$\widetilde{K}_\mu = N_f \frac{\alpha_s}{2\pi} \epsilon_{\mu\nu\rho\sigma} A_\rho^a (\partial_\sigma A_\rho^a - \frac{1}{3} g f_{abc} A_\sigma^a A_\rho^b) . \quad (7)$$

In the effective Lagrangian, the anomaly Eq. (1) takes the form

$$\partial^\mu J_\mu^5 = 2 \sum_a^{0,3,8} c^a v^a + \partial^\mu \widetilde{K}_\mu, \quad (8)$$

with

$$v^a = \frac{1}{4} i \text{Tr} [\lambda^a (U - U^\dagger)] . \quad (9)$$

By substituting Eq. (2) into Eq. (8) one gets

$$2M_N G_1(q^2) + q^2 G_2(q^2) = \lambda + 2M_N \widetilde{G}_1(q^2) + q^2 \widetilde{G}_2(q^2), \quad (10)$$

where

$$\begin{aligned}
\lambda &= \frac{\langle p' | \sum_a^{0,3,8} 2c^a v^a | p \rangle}{\bar{u}(p') i \gamma_5 u(p)} \\
&= \frac{4}{F_\pi} (c_0, c_8, c_3) \begin{pmatrix} \cos \theta_3 & -\sin \theta_3 & \theta_2 \\ \sin \theta_3 & \cos \theta_3 & -\theta_1 \\ \theta_1 \sin \theta_3 - \theta_2 \cos \theta_3 & \theta_1 \cos \theta_3 + \theta_2 \sin \theta_3 & 1 \end{pmatrix} \begin{pmatrix} \frac{g_{\eta' NN}}{m_{\eta'}^2} \\ \frac{g_{\eta NN}}{m_\eta^2} \\ \pm \frac{g_{\pi^0 NN}}{m_{\pi^0}^2} \end{pmatrix} \\
&= \frac{4}{F_\pi} \left\{ g_{\eta' NN} m_{\eta'}^2 [c_0 \cos \theta_3 + c_8 \sin \theta_3 + c_3 (\theta_1 \sin \theta_3 - \theta_2 \cos \theta_3)] \right. \\
&\quad + g_{\eta NN} m_\eta^2 [-c_0 \sin \theta_3 + c_8 \cos \theta_3 + c_3 (\theta_1 \cos \theta_3 + \theta_2 \sin \theta_3)] \\
&\quad \left. \pm g_{\pi^0 NN} m_{\pi^0}^2 (c_0 \theta_2 - c_8 \theta_1 + c_3) \right\}, \tag{11}
\end{aligned}$$

where $\theta_1, \theta_2, \theta_3$ are the mixing angles between π^0 and η , π^0 and η' , η and η' , respectively. The parameter g 's are the coupling constants. In the limit $q^2 \rightarrow 0$, and $q^2 G_2(q^2) \rightarrow 0$, then $\lim_{q^2 \rightarrow 0} q^2 \tilde{G}_2(q^2) = G = F_s(\tilde{m}^2 I - \tilde{m} g_{QNN})$ is the residue of the ghost pole contribution [5], where $I = \frac{g_{\eta' NN}}{m_{\eta'}^2} \cos \theta_3 \pm \frac{g_{\pi^0 NN}}{m_{\pi^0}^2} \theta_2 - \frac{g_{\eta NN}}{m_\eta^2} \sin \theta_3$. (Due to $\pi^0 - \eta - \eta'$ mixing, instead of one η' pole we have to put the singlet combination of η', η, π^0 poles. Thus, we substitute \tilde{m} for $\Delta m_{\eta'}$ in the expression of G . In our numerical calculation we take $\tilde{m}^2 = \Delta m_{\eta'}^2 \approx 0.726 \text{ GeV}^2$.) In the expressions of λ and I , the plus or minus sign is for proton or neutron, respectively. One can re-express Eq. (10) at $q^2 = 0$ as

$$G_1(0) = \tilde{G}_1(0) + \frac{1}{2M_N} (\lambda + G). \tag{12}$$

Let $\Delta \tilde{g} = -\tilde{G}_1(0)$ as in Ref. [5], then one find

$$\Delta \Sigma = \frac{1}{2M_N} (\lambda + G) = \frac{1}{2M_N} [\lambda + F_s(\tilde{m}^2 I - \tilde{m} g_{QNN})]. \tag{13}$$

This is a formula of the quark content in the nucleon spin. Neglecting the higher-order isospin corrections, Eq. (13) can reduce to Eq. (24') in Ref. [5]. (For more details, see Appendix.)

3 Difference between $\Delta\Sigma_p$ and $\Delta\Sigma_n$

To calculate the difference between quark content of the proton ($\Delta\Sigma_p$) and that of the neutron ($\Delta\Sigma_n$), we only take amount into terms that carry the plus and minus signs in Eq. (13),

$$\begin{aligned} \Delta\Sigma_p - \Delta\Sigma_n &= \frac{1}{2M_N} \left\{ 2F_s \widetilde{m}^2 \frac{g_{\pi^0 NN}}{m_{\pi^0}^2} \theta_2 + \frac{2F_\pi g_{\pi^0 NN}}{m_{\pi^0}^2} \left[\sqrt{\frac{3}{2}} \frac{(2m_K^2 + m_\pi^2)}{3} \theta_2 \right. \right. \\ &\quad \left. \left. + \frac{2}{\sqrt{3}} (m_K^2 - m_\pi^2) \theta_1 + \delta m^2 \right] \right\}. \end{aligned} \quad (14)$$

Due to $\theta_2 = -\sqrt{\frac{3}{2}} \frac{\delta m^2}{m^2}$ and $\theta_1 = -\theta_2 \frac{m_{\eta'}^2 + m_\eta^2 - 2m_\pi^2}{2\sqrt{2}(m_K^2 - m_\pi^2)}$, one can re-express Eq. (14) as

$$\begin{aligned} \Delta\Sigma_p - \Delta\Sigma_n &= \frac{1}{2M_N} \left\{ 2F_s \widetilde{m}^2 \frac{g_{\pi^0 NN}}{m_{\pi^0}^2} \theta_2 + \frac{2F_\pi g_{\pi^0 NN}}{m_{\pi^0}^2} \left[\sqrt{\frac{3}{2}} \frac{(2m_K^2 + m_\pi^2)}{3} \right. \right. \\ &\quad \left. \left. - \frac{1}{\sqrt{6}} (m_{\eta'}^2 + m_\eta^2 - 2m_\pi^2) - \sqrt{\frac{2}{3}} (\widetilde{m}^2) \right] \theta_2 \right\} \\ &= \frac{1}{2M_N} \left\{ \frac{2F_\pi g_{\pi^0 NN}}{\sqrt{6} m_{\pi^0}^2} 3m_\pi^2 \right\} \theta_2 \\ &\approx \frac{3F_\pi g_{\pi^0 NN}}{\sqrt{6} M_N} \theta_2 = 2\sqrt{\frac{3}{2}} g_A^3 \theta_2, \end{aligned} \quad (15)$$

where we have made use of Goldberger-Treiman relation, *i.e.* $g_A^3 = \frac{F_\pi g_{\pi^0 NN}}{2M_N}$. It is clear that Eq. (14) reduces to the result of Ref. [5], once neglecting higher-order isospin corrections.

4 Charge symmetry breaking in the pion-nucleon coupling constant and the value of the ghost-nucleon coupling constant

Usually one neglects charge symmetry breaking (CSB), since most of popular mechanisms provides a very small contribution. However, the analysis without this supposition [9] leads to a large magnitude of CSB. The Nijmegen group has recently completed the phase shift analysis of all NN scattering data below $E_{\text{lab}} = 350$ MeV. This is a continuation of the Nijmegen analysis between $0 \sim 30$ MeV. Both

in the pp and np analysis, a low value for the πNN coupling constant was found, indicating a large charge symmetry breaking. They found the following results [10].

$$\begin{aligned} G_{pp\pi^0} &= g_0 , & \frac{G_{pn\pi^+}}{\sqrt{2}} &= g_0 + \frac{3}{2}\Delta g , \\ -G_{nn\pi^0} &= g_0 + 2\Delta g , & \frac{G_{np\pi^-}}{\sqrt{2}} &= g_0 + \frac{3}{2}\Delta g , \end{aligned} \quad (16)$$

where $g_0^2 = \left(\frac{2m_p}{m_c}\right)^2 \times f_0^2 = 13.48$ is $pp\pi^0$ coupling constant, and $\Delta g^2 = g_c^2 - g_0^2 = (G_{pn\pi^+}G_{np\pi^-})/2 - g_0^2 = \left(\frac{m_p+m_n}{m_c}\right)^2 f_c^2 - 13.48 = 0.06$, with g_c^2 the charged coupling constant.

After solving Eq. (16) we find

$$\begin{aligned} g_{pp\pi^0} &= g_0^2 = 13.48 , \\ g_{nn\pi^0} &= G_{nn\pi^0}^2 = 13.61 . \end{aligned} \quad (17)$$

On the other hand, the contribution of the mixing of the π^0 -meson and the ghost pole of the $U(1)$ problem to the pion-nucleon coupling constant is obtained by Kochelev [7]. It is shown that the value of CSB in these constants is defined by the mass difference of d -and u -quark and the value of the ghost-nucleon coupling constant. According to Ref. [7],

$$\frac{g_{nn\pi^0}^2 - g_{pp\pi^0}^2}{2g_{\pi^0 NN}^2} = \frac{g_{QNN}m_\pi(m_d - m_u)}{g_{\pi^0 NN}\sqrt{m_d m_u}} , \quad (18)$$

or

$$g_{QNN} = \frac{(g_{nn\pi^0}^2 - g_{pp\pi^0}^2)\sqrt{m_d m_u}}{2g_{\pi^0 NN}m_\pi(m_d - m_u)} . \quad (19)$$

Using Eq. (17) and $\frac{m_d - m_u}{m_d + m_u} = 0.27 \pm 0.03$ [11], we find the value of the ghost-nucleon coupling constant:

$$g_{QNN} = 3.49 \times 10^{-3} \text{MeV}^{-1} . \quad (20)$$

5 Discussions and conclusions

In this letter we have taken a particular Lagrangian as an effective QCD Lagrangian and have derived a formula of the quark content of the nucleon spin. According to this formula, we have obtained the difference between $\Delta\Sigma_p$ and $\Delta\Sigma_n$.

Now we evaluate the numerical value of $\Delta\Sigma$ and $\Delta\tilde{g}$. The physical masses of mesons are $m_{\eta'} = 958$ MeV, $m_\eta = 549$ MeV, $m_{\pi^0} = 135$ MeV, and the mixing angles determined by Ref. [12] are

$$\theta_1 = -1.7 \times 10^{-2}, \quad \theta_2 = 0.9 \times 10^{-2}, \quad \theta_3 = -0.31. \quad (21)$$

The π -decay constant is $F_\pi = 186.4$ MeV, and the η' decay constant is $f_{\eta'} = 132$ MeV. Using these values, together with the values $g_{\eta'NN} = 6.3$, $g_{\eta NN} = 6.1$, $g_{\pi^0 NN} = 13.55$, one obtains, from Eq. (13) and Eq. (20)

$$\Delta\Sigma = \begin{cases} 0.66 & \text{for proton ,} \\ 0.50 & \text{for neutron ,} \end{cases} \quad (22)$$

and

$$\Delta\Sigma_p - \Delta\Sigma_n = 0.16. \quad (23)$$

Frois and Karliner [13] found the fact that in the lowest order of perturbation theory there is an agreement between the neutron and proton experiments, but when higher-order QCD corrections are taken into account, all of the experimental results converge to the value $G_1(0) \approx 0.30 \pm 0.11$. Using this result one obtains, from Eq. (12) and Eq. (20)

$$\Delta\tilde{g} = \begin{cases} 0.36 & \text{for proton ,} \\ 0.20 & \text{for neutron .} \end{cases} \quad (24)$$

Several comments are now in order. First, neglecting the higher-order corrections, from Eq. (15) one can obtain $\Delta\Sigma_p - \Delta\Sigma_n = 0.03$. This result is much smaller than Eq. (23). It implies that higher-order corrections are non-negligible. Second, another new result of this letter is a detailed calculation of the ghost-nucleon coupling constant. The large value $\Delta m_{\eta'} g_{QNN} \approx 2.97$ makes it unreasonable to assume that $\Delta m_{\eta'} g_{QNN} \ll g_{\eta'NN} \approx 6.3$, as in Ref. [5]. When ghost pole exchange is taken into account in the OBEP (One Boson Exchange Potential) analysis of NN scattering, $g_{\eta'NN}$ approaches to $\sqrt{g_{\eta'NN}^2 - m_{\eta'}^2 g_{QNN}^2}$. Due to large value of $m_{\eta'}^2 g_{QNN}^2$, it implies the great suppression of $g_{\eta'NN}$. This result just agrees with the fact that the

Skyrme model predicts $g_{\eta'NN} \approx 0$ in the large N_c limit. On the other hand, it is enough to prove that Eq. (13) can reduce to Eq. (24') in Ref. [5], neglecting the higher-order isospin corrections. It implies that Eq. (13) possesses more general meaning, comparing with Eq. (24') in Ref. [5].

To summarize, in this letter we have taken a particular Lagrangian, which was introduced to resolve $U(1)$ problem, as an effective QCD Lagrangian and have derived a formula of the quark content of the nucleon spin. The difference between quark content of the proton ($\Delta\Sigma_p$) and that of the neutron ($\Delta\Sigma_n$) is evaluated by this formula. Neglecting higher-order isospin corrections, this formula can reduce to Efremov's results in the large N_c limit.

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Appendix

Neglecting higher-order isospin corrections, Eq. (13) can reduce to Eq. (24') of Ref. [5]. The proof is as follows.

First term of Eq. (13) :

$$\begin{aligned}
& \frac{F_\pi g_{\eta' NN}}{2M_N m_{\eta'}^2} \left[\cos \theta_3 \frac{2m_K^2 + m_\pi^2}{\sqrt{6}} - \frac{2}{\sqrt{3}}(m_K^2 - m_\pi^2) \sin \theta_3 + (m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2) \right. \\
& \quad \left. \times (\theta_1 \sin \theta_3 - \theta_2 \cos \theta_3) \right] + F_s \widetilde{m}^2 \frac{g_{\eta' NN}}{m_{\eta'}^2} \cos \theta_3 \\
& \longrightarrow \frac{F_\pi g_{\eta' NN}}{2M_N m_{\eta'}^2} \left[\cos \theta_3 \frac{2m_K^2 + m_\pi^2}{\sqrt{6}} + \sqrt{\frac{3}{2}}(m_{\eta'}^2 + m_\eta^2 - 2m_K^2) \cos \theta_3 - \frac{2}{\sqrt{3}}(m_K^2 - m_\pi^2) \sin \theta_3 \right] \\
& \longrightarrow \frac{F_\pi g_{\eta' NN}}{2M_N m_{\eta'}^2} \left[\cos \theta_3 \frac{3m_{\eta'}^2}{\sqrt{6}} + \frac{\cos \theta_3 (3m_{\eta'}^2 - 4m_K^2 + m_\pi^2) - 2\sqrt{2}(m_K^2 - m_\pi^2) \sin \theta_3}{\sqrt{6}} \right] \\
& \longrightarrow \frac{F_\pi g_{\eta' NN}}{2M_N} \cos \theta_3 \frac{3}{\sqrt{6}} \longrightarrow \frac{\sqrt{3}}{2M_N} \cdot \frac{F_\pi}{\sqrt{2}} g_{\eta' NN} = \frac{\sqrt{N_f}}{2M_N} f_{\eta'} g_{\eta' NN}
\end{aligned}$$

Second term of Eq. (13) :

$$\begin{aligned}
& \frac{F_\pi g_{\eta NN}}{2M_N m_\eta^2} \left[-\sin \theta_3 \frac{2m_K^2 + m_\pi^2}{\sqrt{6}} - \frac{2}{\sqrt{3}}(m_K^2 - m_\pi^2) \cos \theta_3 + (m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2) \right. \\
& \quad \left. \times (\theta_1 \cos \theta_3 + \theta_2 \sin \theta_3) \right] - F_s \widetilde{m}^2 \frac{g_{\eta NN}}{2m_\eta^2 M_N} \sin \theta_3 \\
& \xrightarrow{f_{\eta'} = \frac{F_\pi}{\sqrt{2}}} \frac{f_{\eta'} g_{\eta NN}}{2M_N m_\eta^2} \left[-\frac{2m_K^2 + m_\pi^2}{\sqrt{3}} \sin \theta_3 - \frac{2\sqrt{2}}{\sqrt{3}}(m_K^2 - m_\pi^2) \cos \theta_3 \right] - \frac{f_{\eta'} g_{\eta NN}}{2M_N m_\eta^2} \sin \theta_3 \sqrt{3} \widetilde{m}^2 \\
& \xrightarrow{\sin \theta_3 = -\frac{2\sqrt{2}(m_K^2 - m_\pi^2)}{3m_{\eta'}^2}} \left[\sin \theta_3 \left(-\sqrt{3} \widetilde{m}^2 - \frac{2m_K^2 + m_\pi^2}{\sqrt{3}} \right) - m_{\eta'}^2 \sin \theta_3 \cos \theta_3 \right] \frac{f_{\eta'} g_{\eta NN}}{2M_N m_\eta^2} \\
& \xrightarrow{f_\eta = f_{\eta'}} \frac{f_\eta g_{\eta NN}}{2M_N m_\eta^2} \left[\sin \theta_3 (1 - \sqrt{3}) m_{\eta'}^2 + \sin \theta_3 (\sqrt{3} (2m_K^2 - m_\eta^2) - \frac{2m_K^2 + m_\pi^2}{\sqrt{3}}) \right] \\
& \xrightarrow{f_\eta g_{\eta NN} = 2M_N g_A^8 / \sqrt{6}} \frac{g_A^8}{\sqrt{6} m_\eta^2} (\sin \theta_3 (1 - \sqrt{3}) m_{\eta'}^2) \\
& \longrightarrow \frac{g_A^8}{\sqrt{6} m_\eta^2} \theta_3 (1 - \sqrt{3}) \sqrt{6} m_\eta^2 \longrightarrow -\frac{1}{\sqrt{2}} g_A^8 \theta_3
\end{aligned}$$

Third term of Eq. (13) :

$$\begin{aligned} & \pm \frac{1}{2M_N} \left\{ F_s \widetilde{m}^2 \frac{g_{\pi^0 NN}}{m_{\pi^0}^2} \theta_2 + \frac{F_\pi g_{\pi^0 NN}}{m_{\pi^0}^2} \left[\sqrt{\frac{3}{2}} \frac{2m_K^2 + m_\pi^2}{3} \theta_2 + \frac{2}{\sqrt{3}} (m_K^2 - m_\pi^2) \theta_1 + \delta m^2 \right] \right\} \\ & \longrightarrow \pm \sqrt{\frac{3}{2}} g_A^8 \theta_2 \quad (\text{See Section 3.}) \end{aligned}$$